Emergent Structure from the Universal Constraint

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Abstract

Given a nontrivial universal constraint on a space of states, we show that admissible states necessarily admit a coarse-grained description. This induces equivalence classes, a refinement structure, and a natural ordering relation. An entropy function defined on these classes provides a monotonic parameterization of the ordering. These structures arise solely from the existence of the constraint and require no prior assumption of dynamics, spacetime, or external time.

1 Setup

Let $\mathcal{S}$ denote the admissible state space defined by

$$C_a \Psi = 0.$$

As established in [1], the existence of such a constraint is necessary for any nontrivial physical structure. We now examine its structural consequences.

2 Constraint-Induced Distinctions

The constraint partitions the space of states into admissible and inadmissible elements. Within the admissible set $\mathcal{S}$, further distinctions arise from the requirement that physically meaningful descriptions be stable under admissibility-preserving transformations.

Admissibility is defined by the constraint condition $C_a \Psi = 0$, as established in [1].

States that differ only by features that are not preserved under such transformations cannot define stable descriptions. This implies that admissible states must be grouped according to their stability under admissibility-preserving transformations.

3 Coarse-Grained Descriptions

We formalize this grouping through an equivalence relation on $\mathcal{S}$.

Let $\sim$ denote an equivalence relation such that

$$\Psi \sim \Psi' \quad \text{if and only if} \quad \Psi \text{ and } \Psi' \text{ are indistinguishable at a chosen descriptive level.}$$

A macrostate is defined as the equivalence class

$$[\Psi] = \{\Psi' \mid \Psi' \sim \Psi\}.$$

Coarse-graining is not a dynamical process, but a descriptive identification of states that are equivalent under admissibility-preserving transformations.

4 Refinement Structure

The equivalence classes admit a natural refinement relation.

Given two macrostates $[\Psi_1], [\Psi_2]$ we define

$$[\Psi_1] \subseteq [\Psi_2]$$

to mean that $[\Psi_1]$ is a refinement of $[\Psi_2]$, corresponding to a more detailed description.

Thus the existence of a constraint implies that admissible states organize into an ordered structure of descriptions at different resolutions.

5 Entropy as an Ordering Parameter

Each macrostate corresponds to a set of admissible states. Let $|[\Psi]|$ denote the size (or, more generally, a measure) of the equivalence class $[\Psi]$.

We define the entropy of a macrostate by

$$n = |[\Psi]|, \qquad S = \log n.$$

Lemma (Monotonicity of Entropy under Refinement).

If $[\Psi_1] \subseteq [\Psi_2]$, then

$$S_1 \leq S_2.$$

Proof. If $[\Psi_1] \subseteq [\Psi_2]$, then $|[\Psi_1]| \leq |[\Psi_2]|$. Since the logarithm is monotonic, it follows that $S_1 \leq S_2$. □

Entropy therefore provides a monotonic label for the refinement ordering of macrostates.

6 Emergent Ordering

The refinement structure together with the entropy function defines an ordering on the space of macrostates.

Sequences of macrostates ordered by refinement admit a monotonic parameterization in terms of entropy. This provides a natural ordering of descriptions without introducing an external time parameter.

The ordering arises entirely from the structure of admissible states under the universal constraint.

7 Summary

A nontrivial constraint on a state space implies that admissible states necessarily admit a coarse-grained description. This induces:

- equivalence classes of states (macrostates),
- a refinement structure on these classes,
- a partial ordering of descriptions,
- and a monotonic entropy function labeling this ordering.

These structures arise without assuming dynamics, spacetime, or a fundamental notion of time.

In subsequent work, we show that additional consistency conditions applied to this ordered structure lead to emergent locality and geometric structure.

References

[1] Diana Haskins, *The Universal Constraint: Definition and Necessity of Constraint*, Zenodo, 2026.
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